Linear Algebra II

16/03/2009, Monday, 09:00-11:00

1 **Gram-Schmidt process**

Consider the vector space of P_3 with the inner product

$$\langle p, q\rangle = p(-2)q(-2) + p(0)q(0) + p(2)q(2).$$

(a) Is the basis $\{1, x, x^2\}$ an orthonormal basis?

(b) Apply Gram-Schmidt process to obtain an orthonormal basis.

(c) Find the coordinates of the polynomial $1 + x + x^2$ in the orthonormal basis obtained above.

2 **Orthogonality**

Consider the vector space of $\mathbb{R}^{2\times 2}$ with the inner product

$$\langle A, B\rangle = \text{trace}(A^T B)$$

where trace(M) denotes the sum of the diagonal elements of the matrix M. Let $S \subset \mathbb{R}^{2\times 2}$ be the subspace of symmetric matrices.

(a) Find a basis for S.

(b) Is this an orthonormal basis? If not, find an orthonormal basis for S.

(c) Find the closest symmetric matrix to the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

3 **Diagonalization**

Solve the initial value problem

$$\begin{aligned} x' &= x - z \\ y' &= y + 2z \\ z' &= x + y + z \end{aligned}$$

with $x(0) = 1$, $y(0) = 1$, and $z(0) = 4$.

4 **Singular value decomposition**

Two matrices $A, B \in \mathbb{R}^{m\times m}$ are called *unitarily equivalent* if there exists an orthogonal matrix $W \in \mathbb{R}^{m\times m}$ such that $A = WBW^T$. Prove of disprove the statement:

Two matrices are unitarily equivalent if and only if they have the same singular values.